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DEVELOPMENT OF A METHOD FOR DEFINING THE TORSIONAL STIFFNESS OF THE FRAME IN THE INITIAL PHASE OF DESIGNING A HEAVY MOTOR VEHICLE

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RESEARCH ARTICLE

ABSTRACT: The conceptual design represents the first concretization of the project task, and its main goal is to define the parameters of the driver's ergo-sphere, external dimensions, weight, and performance of the heavy motor vehicle, as well as its stylistic indicators, necessary for further work on the project.

As is known, the parameters of the frame are not known in the initial phase of designing a heavy motor vehicle, so in this paper, an attempt was made to define the required torsional stiffness of the frame based on its transverse vibrations. For this purpose (with the introduced hypothesis), a simplified geometric, physical, and mathematical model of the frame was used, whose random transverse vibrations will be analyzed using some methods of mathematical statistics and the three-variable Fourier transformation.

KEY WORDS: *Heavy motor vehicle, frame, transverse vibrations, torsional stiffness, three-variable Fourier transform*

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PRILOG RAZVOJU METODE ZA DEFINISANJE TORZIONE KRUTOSTI OKVIRA U FAZI IZRADE IDEJNOG PROJEKTA TERETNOG MOTORNOG VOZILA

REZIME: Idejni projekat predstavlja prvu konkretizaciju projektnog zadatka, a osnovni cilj mu je definisanje parametara ergosfere vozača i spoljašnjih dimenzija, masa i performansi teretnog vozila, kao i njegovih stilskih pokazatelja, neophodnih za dalji rad na projektu.

Kao što je poznato, parametrii okvira nisu poznati u početnoj fazi projetovanja teretnog vozila, pa je, u ovom radu učinjen pokušaj definisanja potrebne torzione krutosti okvira, na bazi njegovih poprečnjih vibracija.U te svrhe (uz uvedenu hipodezu) je korišćen uprošćeni geometrisjki, fizički i matematički model okvira, čije će slučajne poprečne vibracije biti analizirane uz korišćenje nekih metoda matematičke statistike i i troparametarska Furijeova transformacija..

KLJUČNE REČI: *Teretno vozilo, okvir, poprečne vibracije, torziona krutost troparametarska Furijeova transformacija*

DEVELOPMENT OF A METHOD FOR DEFINING THE TORSIONAL STIFFNESS OF THE FRAME IN THE INITIAL PHASE OF DESIGNING A HEAVY MOTOR VEHICLE

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INTRODUCTION

A heavy motor vehicle defined by the project task is developed in further design phases, where creative and intuitive approaches that played a significant role in the project task development give way to logical and objective factors, calculations, measurements, shaping, evaluations of production and technological capabilities, etc. [1,2].

The conceptual design represents the first concretization of the project task, and its main goal is to define the parameters of the driver's ergo-sphere and the external dimensions, mass, and performance of the cargo vehicle, as well as its stylistic indicators necessary for further work on the project [1,2] heavy motor vehicle.

This paper will discuss the development of a method for defining the torsional stiffness of the frame based on its transverse vibrations. Therefore, the hypothesis is introduced: "Transverse vibrations of the vehicle frame can be used as a parameter for defining torsional stiffness if their quantitative values do not change drastically across the frame's surface, in case of rigorous disturbances (forces, torques)".

The assumption is that the project task defines the need to design a heavy motor vehicle for the market with a total mass of 11,000 kg and a payload capacity of 4,000 kg, with dimensions (length * width * height, mm): 6400 * 2500 * 3600, with a short cab. The engine is positioned at the front, and the vehicle has all-wheel drive.

The designed vehicle must withstand rigorous operating conditions. Other parameters are not mentioned here because they are not of particular importance for defining the torsional stiffness of the frame or investigating its transverse vibrations. For illustration purposes, Figure 1a shows the silhouette of the heavy motor vehicle defined by the project task [1].

Based on the analysis of existing analogous vehicles and the requirements regarding the dimensions of the newly designed vehicle, it was assessed that the frame length could be 6100, mm, and its width 800, mm. The structure would be ladder-type, as shown in Figure 1b) for illustration purposes [3,4].

The dimensions of the corresponding frame profiles should be determined based on their flexural and torsional stiffness. It is emphasized that in practice, an approximate relationship between the stiffness of the springs and the torsional stiffness of the frame is defined [1]. However, since the parameters of the vehicle suspension system are not known in this design phase, the desired torsional stiffness of the frame cannot be precisely defined...

Therefore, it was deemed appropriate to calculate the equivalent height of the frame for the analysis of transverse vibrations based on the mass balance. Namely, it is recommended in [1,2] that the frame of the vehicle should account for 11-15% of its mass. Since it is a lighter heavy motor vehicle, it will be assumed that this share is 12%, which means that the mass of the frame is 840 kg. Bearing this in mind, the following text will discuss the possibilities for analyzing the vibrations of the vehicle frame in this design phase.

1. METHOD

As already mentioned, the concept of a frame and its length and width have been adopted. Now, based on the required torsional stiffness of the frame, it is necessary to define the dimensions of the longitudinal and transverse profiles using some calculation methods, most commonly finite element methods [5,6]. However, as mentioned before, the bending and torsional stiffness, as well as precise external loads, are not known in this design phase, so the application of the mentioned method is not possible with satisfactory reliability.

Considering the aforementioned, as well as the introduced hypothesis, it was deemed appropriate to develop a procedure based on the analysis of transverse vibrations of the vehicle frame. It should be noted that in this design phase, a large number of frame parameters are unknown, so some of them must be obtained through the study of simpler models. It was considered appropriate to idealize and observe the frame as a homogeneous plate of the adopted length and width, with an unknown thickness [7], as shown in Figure 1c). The plate undergoes transverse vibrations under the influence of disturbing forces at the connection points of aggregates, and systems to the frame.

Since the length and width of the frame have been adopted, it is necessary to define the plate thickness based on which the required torsional stiffness of the frame will be calculated. Assuming that the frame is made of steel, the problem was solved by calculating the equivalent plate thickness of 23, mm based on its mass.

To analyze the transverse vibrations of the frame model, it was necessary to define dynamic excitations. Considering that not all excitations are known in this project phase (uneven engine operation, road irregularities, tire non-uniformity, etc.), it was deemed appropriate to analyze vibrations under conditions of short-term intensive braking [8,9]. Based on experience, an impulse shape was chosen with the presence of random changes, as illustrated in Figure 1d).

For further analysis, it was adopted that the engine is supported by the frame at four points, and the cabin is also supported at four points [1]. The cargo box is supported at eight points, but for the sake of simplifying the problem, it is assumed that it is attached to the frame at four points. As for the springs, each of them is connected at two points, but for the same reasons as the cargo box, it is assumed that they are connected at one point each [1]. The illustration of the connecting points is shown in Figure 1d), and the empirical coordinates of the connecting points are given in Table 1.

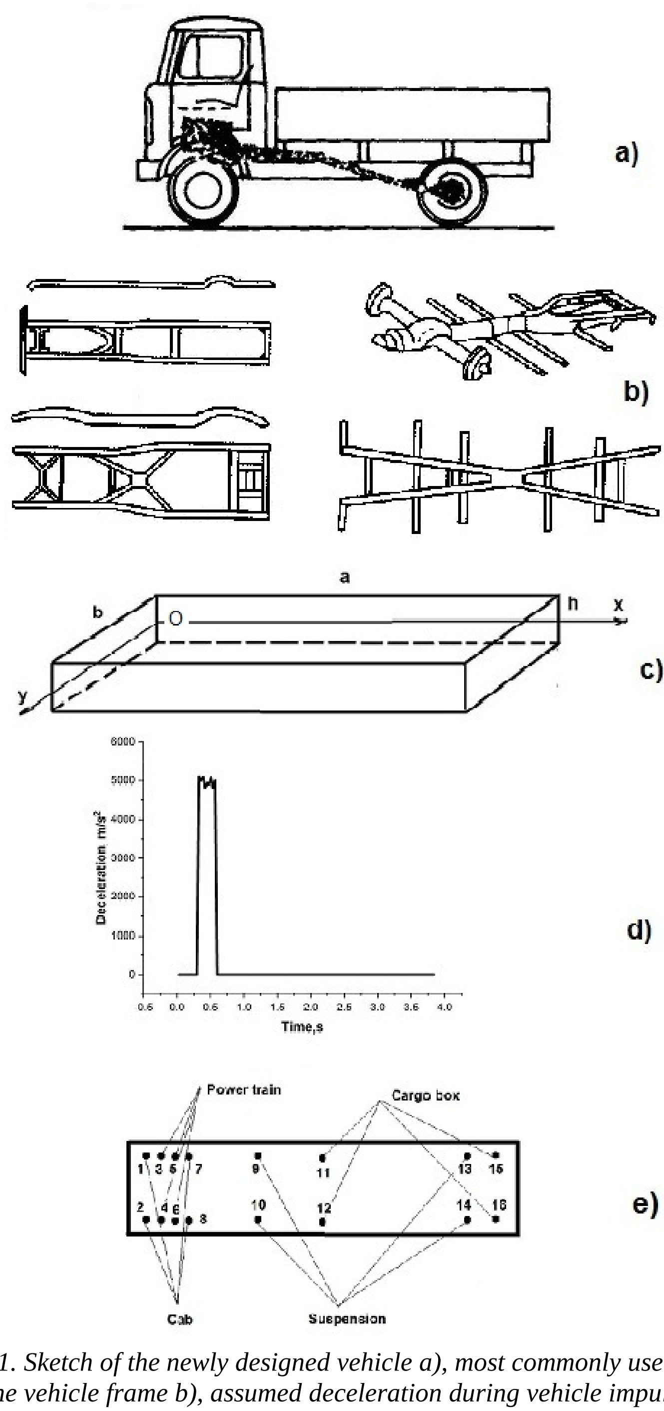

Figure 1. Sketch of the newly designed vehicle a), most commonly used structural solutions for the vehicle frame b), assumed deceleration during vehicle impulse braking c), and equivalent connection points

Table 1. Coordinates of conecting points

	Coordinate x, mm	Coordinate y, mm
1	365	40
2	365	760
3	560	40
4	560	760
5	1375	40
6	1375	760
7	1496	40
8	1496	760
9	1600	40
10	1600	760
11	3400	40
12	3400	760
13	5015	40
14	5015	760
15	5050	40
16	5050	760

To determine the driving forces, it was necessary to calculate the characteristic masses of the aggregates. This was done using statistical data on the percentage participation of aggregate masses in the vehicle mass, as well as based on recommendations on the size of the supported mass [1,2,8,9].

In addition to the mass of the aggregates and systems, it was necessary to calculate the distance between the supports along the length of the vehicle frame (which was done using data from Table 1) and define the height of the aggregate's center of gravity relative to the upper edge of the frame [1]. Approximate data is given in Table 2.

Table 2. Longitudinal distance of connecting points, the height of the center of gravity relative to the frame, and mass of aggregates and systems

	Distance, mm	Height of center of gravity, mm	Mass, kg
Power train	1010	200	962
Cab	1040	800	578
Cargo box	1915	850	890
Suspwnded mass	3600	1200	8070

Inertial force due to braking is given by the expression [7,8]:

$$F_i = m_i a \ , \tag{1}$$

where:

- a - acceleration defined by Figure 1d),
- m_i - corresponding mass (powertrain group, cabin, cargo box, suspended mass).

We will assume that the center of gravity of the aggregate and system is located at the midpoint of the longitudinal distance, so in that case, the static load (Fst) is equal on all supports and amounts to one-fourth of the gravitational force. The static force increases on the front supports and decreases on the rear supports during vehicle braking. The magnitude of the force change due to braking is given by the expression [8,9]:

$$\Delta Z = \frac{F_i h_{ti}}{4L_i} , \quad (2)$$

where:

- F_i - inertial force due to braking, defined by expression (1),
- h_{ti} - height of the center of gravity of the corresponding mass from Table 2,
- L_i - longitudinal distance between supports from Table 2,
- \+ refers to the front supports, and - refers to the rear supports.

Based on expressions (1 and 2), the force on each support is calculated [8,9]:

$$F = F_{st} \pm \Delta Z , \quad (3)$$

Transverse vibrations of the elastic plate are described by a partial differential equation.

The following assumptions were made during its evaluation [7,10]:

- the thickness of the plate is small compared to its dimensions,
- the mid-plane of the plate does not deform and remains as the neutral plane after deformation or bending,
- displacements of the mid-surface of the plate are small compared to the thickness of the plate,
- the influence of transverse shear deformation is neglected, resulting in the fact that the planes normal to the mid-surface before deformation remain normal to the mid-surface even after deformation or bending,
- transverse normal deformation can be neglected under transverse loading, as well as the corresponding stress,
- the cross-section of the frame is rectangular and constant along its length, and
- the material of the frame is homogeneous.

Since the evaluation of the partial differential equation describing the transverse vibrations of an elastic plate is detailed in [7], it will not be done here, but only its final form will be presented.

Based on the introduced assumptions, the forced transverse vibrations of the plate are described by the partial differential equation [7]:

$$D\left(\frac{\partial^4 u}{\partial x^4} + 2\frac{\partial^4 u}{\partial x^2 \partial y^2} + \frac{\partial^4 u}{\partial y^4}\right) + \rho h \frac{\partial^2 u}{\partial t^2} = f(x, y, t) , \quad (4)$$

where:

- $u = u(x, y, t)$ - transverse vibrations of the frame,
- x - coordinate along the length of the frame,

- *y* - coordinate along the width of the frame,
- *f(x, y, t)* - disturbance transverse force (excitation function),
- *t* - time.

The value of D is given by the expression:

$$D = \frac{Eh^3}{12(1-\upsilon^2)} \quad , \tag{5}$$

where:

- *E* - Young's modulus,
- ν - Poisson's ratio, and
- *h* - plate thickness.

As known [7,10,11], to find the general solution of the partial differential equation (4), it is necessary to know the boundary and initial conditions.

In this specific case, all edges are free (moments and shear forces are equal to zero), and the vibrations and their velocities are equal to zero at the initial moment [7].

Mathematically, these conditions are defined by the equations:

$$\begin{aligned}
&M_x = -D(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\upsilon \partial y^2}) = 0 : x = 0 \\
&V_x = Q_y - D[\frac{\partial^3 u}{\partial x^3} + (2-\upsilon)\frac{\partial^3 u}{\partial x \partial y^2}] : x = 0; Q_y = 0 \\
&M_x = -D(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\upsilon \partial y^2}) = 0 : x = a \\
&V_x = Q_y - D[\frac{\partial^3 u}{\partial x^3} + (2-\upsilon)\frac{\partial^3 u}{\partial x \partial y^2}] : x = a; Q_y = 0 \\
&M_y = -D(\frac{\partial^2 u}{\partial y^2} + \upsilon \frac{\partial^2 u}{\partial x^2}) = 0 : y = 0 \\
&V_y = Q_x - D[\frac{\partial^3 u}{\partial x^2 \partial y} + (2-\upsilon)\frac{\partial^3 u}{\partial y^3}] : y = 0; Q_x = 0 \\
&M_y = -D(\frac{\partial^2 u}{\partial y^2} + \upsilon \frac{\partial^2 u}{\partial x^2}) = 0 : y = b \\
&V_y = Q_x - D[\frac{\partial^3 u}{\partial x^2 \partial y} + (2-\upsilon)\frac{\partial^3 u}{\partial y^3}] : y = b; Q_x = 0 \\
&u(x, y, 0) = 0 \\
&u'(x, y, 0) = 0.
\end{aligned} \tag{6}$$

The disturbance force represents the sum of dynamic forces at the supports, i.e.:

$$f(x,y,t)=\sum_{i=1}^{16}F_i(t)\,, \tag{7}$$

where the force F_i(t) is defined by the expression (3) calculated at each support.

The integral of the partial differential equation (4), with the boundary, initial conditions (6), and disturbance force (7), can only be sought in the case of harmonic excitation (and not without difficulties), so an attempt was made to solve it using the Wolfram Mathematica 13.2 software [11]. However, this software allows solving partial differential equations up to the second order, so the problem had to be solved numerically [12] using the finite difference method.

The author developed software for solving the partial differential equation (4) using the finite difference method, with boundary, initial conditions (6), and disturbance force (7), in Pascal. It should be noted that in the case of numerical solving of partial differential equations, sometimes it is necessary to introduce additional boundary and initial conditions [11]…

The dynamic simulation was performed for a steel frame structure with the following data: E=2.1*105, N/mm^2; ρ = 8*10-6, kg/mm^3; υ =0.3; n_x=128; n_y=128; n_t=128; h_x=47.65, mm; h_y=6.25, mm; h_t=0.03, s.

Since the transverse vibrations of the frame depend on three parameters, it is necessary to apply 4D graphics for their graphical representation, which is associated with great difficulties. Therefore, for illustration purposes, partial results for the center of gravity planes: xOt (y=const) and yOt (x=const) are shown in Figures 2 and 3, as it was possible to do using commercial 3D graphics.

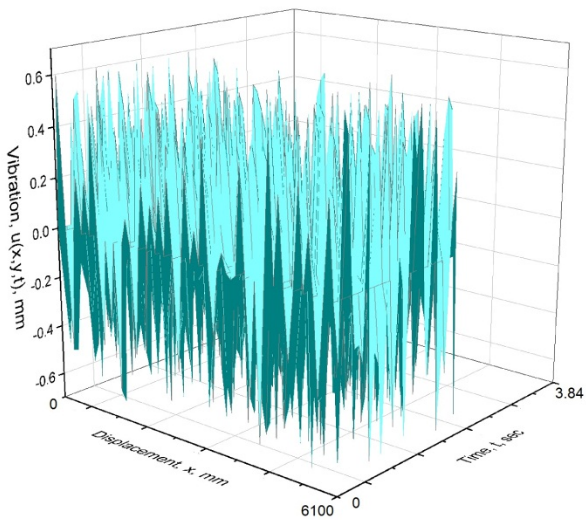

Figure 2. Transverse vibrations of the truck frame in the longitudinal center of gravity plane (xOt)

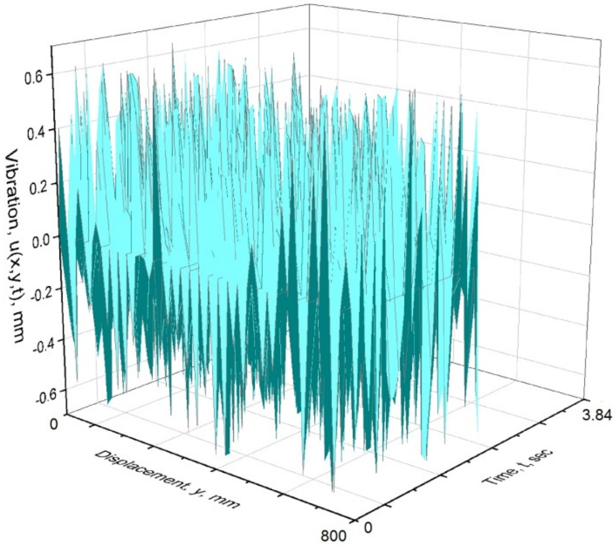

Figure 3. Transverse vibrations of the truck frame in the lateral center of gravity plane (yOt)

Due to the limitations imposed by 3D graphics, the results are partially shown only for the center of gravity of the frame. It was deemed appropriate to calculate the RMS values of vibrations for points with coordinates: 5, 50 and 95% of the length, and 5, 50, and 95% of the width of the frame, for further analysis. The obtained results are shown in Table 3.

Table 3. RMS transverse vibrations at characteristic points of the frame

Coordinates (%*a; %*b): a-length, b-width	RMS*10^{-1}, mm
5 ; 5	3.661
5 ; 95	3.809
50 ; 5	3.839
50 ; 95	3.787
95 ; 5	3.818
95 ; 95	3.885

For a more detailed analysis of the transverse vibrations of the vehicle frame, it was deemed appropriate to perform a frequency analysis [13-16]. Since the transverse vibrations of the used frame model depend on three parameters (coordinates x, y, and time t), it is obvious that a three-variable Fourier transform must be performed [11,13-16], and based on it, the magnitudes and phase angles of the spectrum are calculated.

The author developed software for calculating the 3D Fourier transform and parameters of the mentioned spectrum in Pascal. Using that software, the magnitudes and phase angles of the 3D Fourier transform were calculated.

It should be noted that the spectra, in this case, depend on three parameters (frequency in x direction, frequency in y direction, and frequency in time domain t), which makes their graphical representation difficult, as it requires 4D graphics. Therefore, it was deemed appropriate to only show the spectra in the longitudinal and transverse center of gravity planes, in Figures 4-6.

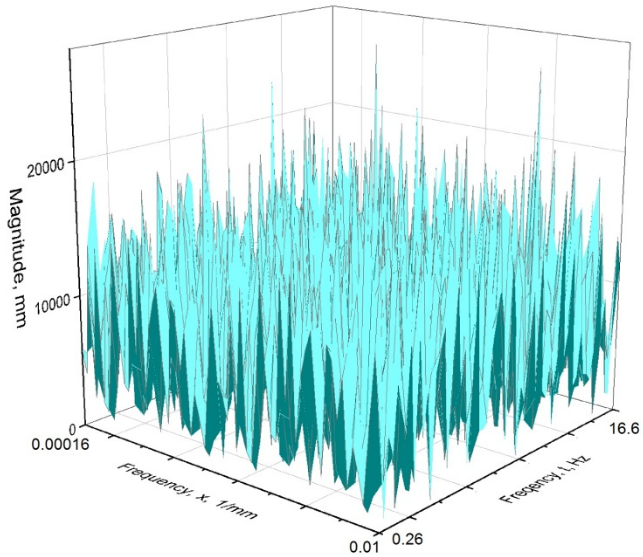

Figure 4. The spectra magnitudes of the heavy motor vehicle frame in the longitudinal center of gravity plane (xOt)

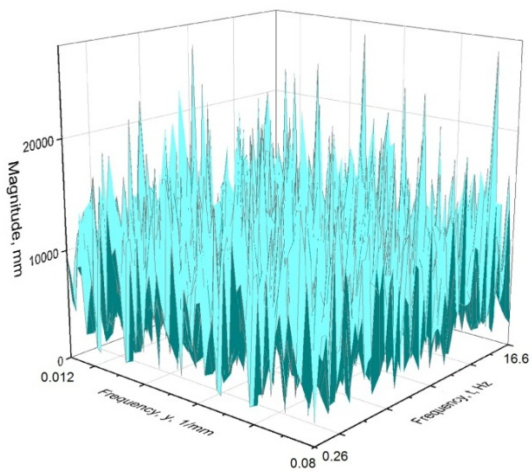

Figure 4. The spectra magnitudes of the heavy motor vehicle frame in the lateral center of Gravity plane (yOt)

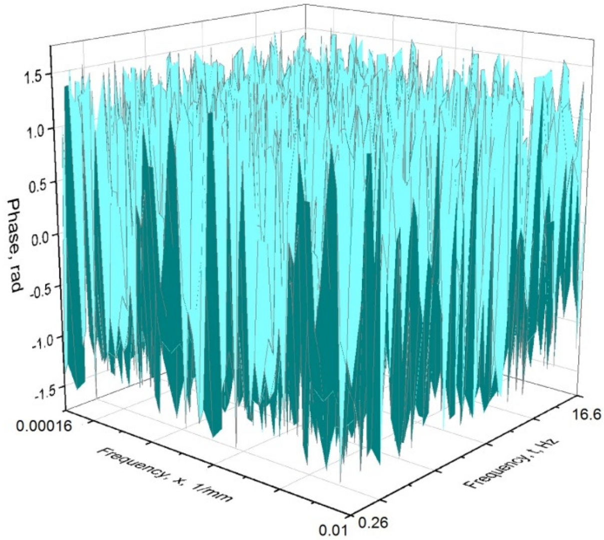

Figure 6. The phase angles of the heavy motor vehicle frame in the longitudinal center of gravity plane (xOt)

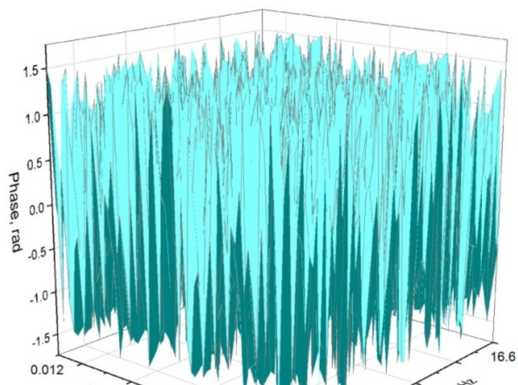

Figure 7. The phase angles of the heavy motor vehicle frame in the lateral center of gravity plane (yOt)

2. DATA ANALYSIS

From Figures 2 and 3, it can be seen that the transverse vibrations for the center of gravity planes of the vehicle frame depend on the position along the length, width, and time. Braking forces are of impact and stochastic nature, which falls into very rigorous conditions in vehicle dynamics [8,9]. More information about the change in vibrations across the surface of the vehicle frame can be found in Table 3.

By analyzing the RMS data from the mentioned table, it can be determined that the transverse vibrations are small (which is desirable from the perspective of vehicle frame construction) and that the RMS varies across the surface of the observed frame model, which can be explained by the fact that the excitation functions had impact and random characteristics and acted at 16 points. However, these differences are small, indicating that the frame behaves stably under the given conditions, confirming the introduced hypothesis. This allows the defined frame parameters to be used in further frame design. More specifically, the calculated equivalent plate thickness can be used to calculate torsional stiffness, and the used positions of the force application points can serve as guidance for the connection of aggregates and vehicle systems.

From Figures 4-7, it can be observed that random vibrations spread along the length and width of the frame with different amplitudes of modulus and phase angles. This is by the data analysis in the x, z, t space, so there will be no further discussion about it here, and the amplitude and frequency of harmonics depend on the design parameters of the frame and the time excitation...

Considering Parseval's theorem, it was deemed unnecessary to calculate RMS values, as is the case with Table 3.

It should be noted that the number of points and the integration step have ensured the reliability of the results in terms of frequency: x (0.00016 - 0.1, 1/mm), y (0.012 - 0.08, 1/mm), and t (0.26 - 16.6 Hz)), which is acceptable from the perspective of frame vibrations in this design phase [18]. In the later stages of the project, when the final structure of the vehicle frame is adopted, research should be conducted with a larger number of points using the finite element method, as existing software allows for automatic mesh generation [19].

It should also be noted that there are no explicit procedures for calculating errors in spectral analysis for 3D Fourier transform, as in the case of 1D Fourier transformation [17]. Bearing this in mind, as well as the fact that this study aims to illustrate the potential of using 3D frequency analysis in investigating transverse vibrations of vehicle frames in the initial design phase, the analysis of statistical errors was not specifically performed...

In the following text, it will be shown how the required torsional stiffness can be calculated based on the defined length, width of the frame, and equivalent plate thickness. Namely, according to [17], there is a known relation that defines the torsional stiffness of a rectangular cross-section:

$$C_t = \beta\, h\, b^3\, G$$
$$G = \frac{E}{2(1+\upsilon)}, \quad (8)$$

where:

- E - Young's and Poisson's coefficients (whose values are given in the previous text),
- β - a parameter that depends on the ratio h/b (the value of this parameter is usually defined for ratios greater than 1; since this is not the case for the frame, interpolation should be performed for smaller values, usually linear).

Using equation (8), the required torsional stiffness of the frame can be calculated, which will not be done here.

3. CONCLUSION

The developed procedure, based on the analysis of transverse vibrations of heavy vehicle frames, allows for the determination of the necessary parameters, including torsional stiffness.

The simplified plate-like frame model provides enough opportunities to verify the connection points of the aggregates in the conceptual design phase of the vehicle frame.

In further development of the project, based on the approximately defined parameters of the vehicle frame, more detailed calculations can be performed, potentially using the finite element method.

The conducted analyses have shown that the use of 3D Fourier transformation is useful in analyzing the transverse vibrations of vehicle frames in the initial design phase.

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